Stability Analysis of Interfacial Waves in Hall-Héroult Process: Feasibility Study

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Abstract

The occurrence of fluid interfacial instabilities due to gravity waves, modified by intense electric/magnetic fields, is one of the important problems in the Hall-Héroult reduction cells. Several parameters affect the flow stability, such as non-uniformity of the walls, density difference, fluid layer thicknesses, wall friction coefficient, magnitude of different forces, etc. In order to analyze the interface stability, first, one can use a two-layer thin-film fluid flow model inside a simplified uniform 2D geometry, while considering the effects of electro-magnetic fields as a source term in the momentum equation. In this study, we explore the possibility of a stability analysis of interfacial waves in Hall-Héroult process by applying a weighted residual model to find the marginal stability of the system at the long-wave length limit. Finally, we numerically analyze the effects of non-uniform lower walls on the convective instability of the interface. The results show that the proposed model can be used to predict the effects of different geometrical parameters as well as operational ones on the stability of cryolite-aluminum interface in the Hall-Héroult process.

Keywords: Hall-Héroult reduction cells, stability analysis, two-phase flow, two-layer model.

1. Introduction

Hall-Héroult process is one of the major industrial process for smelting aluminum. This process is very complex due the presence of hydro-, electro-, magnetohydro-dynamic features, as well as electro-chemical effects. One important problem is the occurrence of fluid interfacial instabilities within the Hall-Héroult reduction cells, due to gravity waves modified by intense electric/magnetic fields. There are many parameters that can affect the stability of the flow, e.g., magnitude of various forces, density difference, fluid layer thicknesses, wall friction coefficient, etc. We simplify the magnetic field and the current that passes through the layers in order to simplify the problem and examine the feasibility of stability analysis of interfacial waves in Hall-Héroult process.

Two-layer, shallow fluid flow inside a simplified 2D geometry with two Newtonian fluids (Aluminum and Cryolite) can be considered as a powerful tool to capture the interfacial instability [1]. Here, two fluids can be assumed immiscible, with a small density difference. The momentum and continuity equations can be developed for both fluids.

Bojarevics has developed a model considering a variable cathode height, where the effects of fluid height on the horizontal current density and wave development was studied [3]. Compared to the previous works on the Hall-Héroult process modeling flows [4, 5, 6, 7], the contribution of this work will be to develop a novel, semi-analytical model to deliver the condition of the instability of the system while taking into account magnetic effects with non-uniform boundary conditions. In particular, non-uniform boundary conditions at the bottom of the flow geometry (cathode block surface) will be considered. A weighted residual model (WRM) [2] approach will be applied to the system to consider weak inertial effects on the flow stability [1]. The results will deliver in particular the threshold for the operating conditions for the Hall-Héroult process, in which fluid interfacial instabilities are avoided. The model will also provide an understanding how the interfacial perturbation may grow, versus the dimensionless groups that govern the flow.

2. Modeling

Figure 1 illustrates a simple schematic of the aluminum production cell. In this system, several parameters affect the instability of the interface between the aluminium and cryolite layers such as the magnetic field and the current that passes through the layers, the non-uniform geometry, two different fluids with different density, etc.

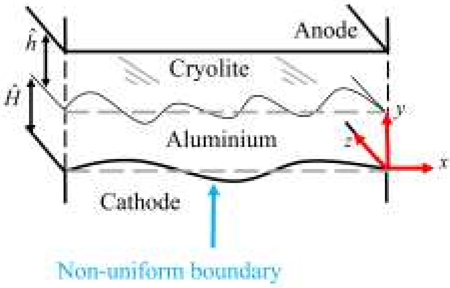

Figure 1. Schematic of an aluminium production cell.

Assuming the electro-magnetic force is dominant in the aluminium layer, we simplify the problem to a system of two layers of fluids in a non-uniform geometry.

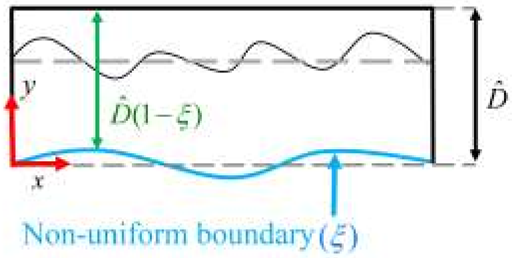

Figure 2. Flow configuration in a non-uniform geometry.

Figure 2 shows schematically the flow configuration in the vertical (*x, y*) plane. Here, the upper wall is uniform and the lower wall is considered non-uniform. The local height of the cell varies in *x* direction as $\hat{D}(1-\xi)$, in which $\hat{D}$ is the height of the imaginary uniform geometry and ξ is the non-uniformity function of the lower wall. We consider two Newtonian fluids with different densities $(\hat{\rho}_1,\hat{\rho}_2)$ and viscosities $(\hat{\mu}_1,\hat{\mu}_2)$. Layers 1 and 2 are taken to correspond to the lower and upper layer, respectively.

We can write the momentum and continuity equations for each layer in the *x* and *y* direction as:

$$\hat{\rho}_1(\frac{\partial \hat{u}_1}{\partial \hat{t}}+\hat{u}_1\frac{\partial \hat{u}_1}{\partial \hat{x}}+\hat{v}_1\frac{\partial \hat{u}_1}{\partial \hat{y}})=-\frac{\partial \hat{p}_1}{\partial \hat{x}}+\hat{\mu}_1\left[\frac{\partial \hat{u}_1{}^2}{\partial \hat{x}^2}+\frac{\partial \hat{u}_1{}^2}{\partial \hat{y}^2}\right]+\hat{F}, \quad (1)$$

$$\hat{\rho}_2(\frac{\partial \hat{u}_2}{\partial \hat{t}}+\hat{u}_2\frac{\partial \hat{u}_2}{\partial \hat{x}}+\hat{v}_2\frac{\partial \hat{u}_2}{\partial \hat{y}})=-\frac{\partial \hat{p}_2}{\partial \hat{x}}+\hat{\mu}_2\left[\frac{\partial \hat{u}_2{}^2}{\partial \hat{x}^2}+\frac{\partial \hat{u}_2{}^2}{\partial \hat{y}^2}\right], \quad (2)$$

6. References

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