

# Multi-Particle Sedimentation under Vibration

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## Abstract

In the aluminum smelting process in production of aluminum, the anodes used are formed through compaction of a paste composed of coarse particles of petroleum coke and binder matrix. The latter is a non-Newtonian material. One of the rheological features of the binder is the presence of a yield stress, which is the focus of our work. One of aspect of the complicated process of vibro-compaction can be seen as sedimentation of numerous particles (coke) in a yield stress material (binder), through applying vibration to the container. In this work, the effects of the vibration of container on the sedimentation of multiple particles in yield stress fluids are numerically studied, and the results are compared with the Newtonian counterparts. It is found that in Newtonian fluids vibration of the container does not necessarily cause the particles to fall faster. Indeed, the vibration may cause the particles to fall slower at higher frequencies. In yield stress fluids, vibration of the container may have a significant effect on the speed of the sedimentation of the particles in a way that higher frequencies cause the particles to fall faster. This effect becomes more pronounced in yield stress fluids with higher yield stresses.

**Keywords:** Aluminum smelting process; vibro-compaction; yield stress; numerical simulation.1

## 1. Introduction

Sedimentation of the particles involved in many industrial projects including slurry flows, fluidization, etc. Particles sedimentation under vibration of the container is one of the main steps of making anode in the process of the production of aluminum (which we explained in the abstract). Numerous researches have been focused on the particles sedimentation in yield stress fluids (e.g. see [1-3]), but to best of our knowledge there is no study related to the effect of the vibration of the container on the sedimentation of the particles.

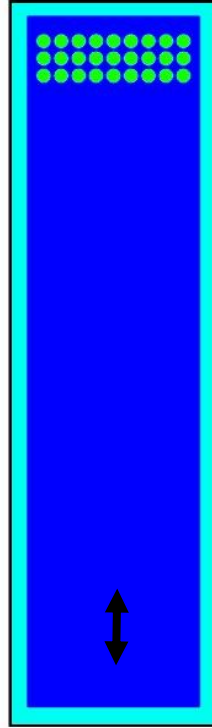
## 2. Problem description and results

The system that we consider has been scaled compared to the industrial application. We consider the problem of many particles sedimentation in Newtonian fluids as well as yield stress fluids in container that oscillates vertically with:

$$y = 0.04 \sin(2\pi f t) \text{ (cm)} \quad (1)$$

where:  $y$  position of the bottom of the container,  $cm$   
 $f$  frequency of oscillation of the container,  $1/s$   
 $t$  time,  $s$

Schematic of the particles and their initial position is shown in Figure 1. The channel is assumed to have a width of 1 (cm) and a height of 4 (cm). Initial position of the particles is chosen in a



**Figure 1. Initial position of the particles.**

way to have their center of mass at height of 3.85 (cm) (see Figure 1). The distance between the centers of each two particles is chosen to be 0.1 (cm). Fluid density is  $\rho_f = 1 (g/cm^3)$ , and kinematic viscosity of the fluid is assumed to be  $0.01 (cm^2 / s)$ . Solid density is  $\rho_s = 1.25 (g/cm^3)$ . Diameter of the particles are  $D = 0.07 (cm)$ . The whole domain and the particles are initially at rest and the container start to oscillate at  $t = 0 (s)$ , at different frequencies.

Yield stress fluid during this study is assumed to be the Bingham model which is the most widely used model for this kind of fluids. Such materials become yielded and flow when a certain yield stress is exceeded. For any stress lower than this critical stress the material is un-yielded. Constitutive law of the Bingham model can be written as [4]:

$$\begin{cases} \tau_{ij} = 2 \left( \frac{\tau_y}{\dot{\gamma}} + \mu_p \right) d_{ij} = 2\eta(\dot{\gamma})d_{ij} & \text{for } \tau_{ij} \geq \tau_y \\ 2d_{ij} = 0 & \text{for } \tau_{ij} < \tau_y \end{cases} \quad (2)$$

where:  $d_{ij}$  rate of deformation tensor  
 $\tau_y$  yield stress  
 $\mu_p$  plastic viscosity  
 $\dot{\gamma}$  deformation rate ( $\dot{\gamma} = \sqrt{d_{ij}d_{ij}}$ )  
 $\eta(\dot{\gamma})$  apparent viscosity of the Bingham model

Detailed descriptions of our numerical method and its verifications are presented in our previous publications [5, 6].

Main dimensionless parameters involved in this study are Archimedes number:

$$Ar = \frac{a_g D^3 (\rho_s - \rho_f)^2}{\mu^2} \quad (3)$$

where:  $a_g$       gravitational acceleration,  $cm/s^2$   
 $\mu$               dynamic viscosity of the fluid,  $g/(cm \cdot s)$

and dimensionless yield stress:

$$Y_g = \frac{\tau_y}{a_g D (\rho_s - \rho_f)} \quad (4)$$

where:  $\tau_y$       yield stress of the fluid,  $g/(cm \cdot s^2)$

Figures 2 – 4 show Y-location of the center of gravity of the particles for the Newtonian case and yield stress fluids (Bingham model cases with  $Y_g = 0.02$  and  $Y_g = 0.05$ ), respectively, versus time. It should be noted that in this study, the Archimedes number is assumed to be  $Ar \square 210$ . As it is shown in Figure 2, when the fluid is Newtonian, the speed of particles settling is approximately independent of shaking the container. In fact, in the Newtonian case, higher frequencies of oscillation of the container, not only does not forces the particles to fall faster, it causes the particles to fall slower! We are shaking the container up and down. Moving the container up has an adverse effect on the particles and causes a decrease in the speed of falling of the particle, and moving the container down helps the particle to fall faster. Due to the direction of the movement of the particles, it seems that in the Newtonian case the increasing effect of the container on the speed of particles is less than its decreasing effect. So, vibrating the container with higher frequencies, case a more decrease on the speed of falling of the particles.

Unlike the Newtonian case, an increase in the frequency of the oscillation in the yield stress fluid causes the particles to fall faster and when the Bingham number is higher, the effect of the oscillation on the speed of falling of the particles is more pronounced (Figures 3 and 4). Probably the reason is that the mechanism of the effect of vibrating the container in the case of Bingham fluid is different from the Newtonian case. It seems that the most important effect of vibrating the container is breaking the structure of the fluid. It helps the particles to overcome the yield stress. Overcoming the yield stress of the particles obviously helps the particles to settle easier.

Figures 5 and 6 show the effect of the fluid type on the falling of the particles in a stationary container and an oscillating container, respectively. As can be when the container is at rest (Figure 5), the fluid properties have a great impact on the speed of falling of the particles, while at high values of frequencies (for example  $f=4(1/s)$  in Figure 6) all of the fluids have approximately the same effect on the falling of the particles. It is due to the fact that high frequencies of oscillation of the container, cause the internal structures of the Bingham fluid to break up and therefore vanishes the yield stress of the fluid.

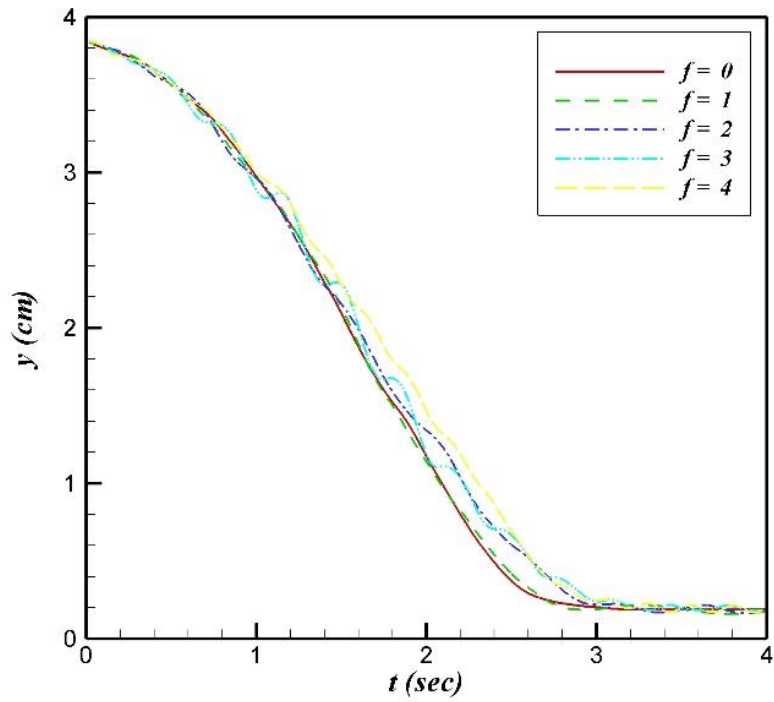


Figure 2. Y-location of the center of gravity of the particles for the Newtonian case.

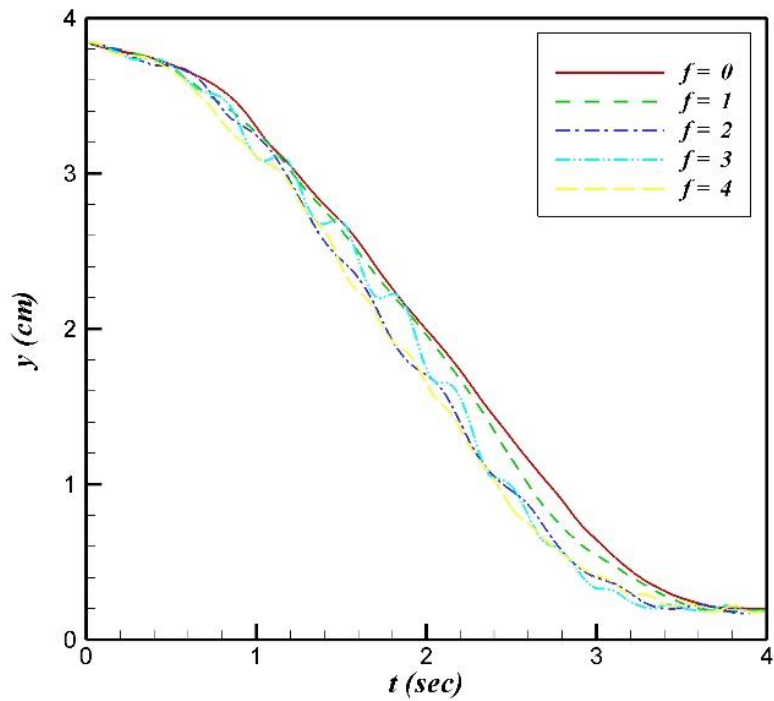


Figure 3. Y-location of the center of gravity of the particles for the yield stress fluid ( $Y_g=0.02$ ).

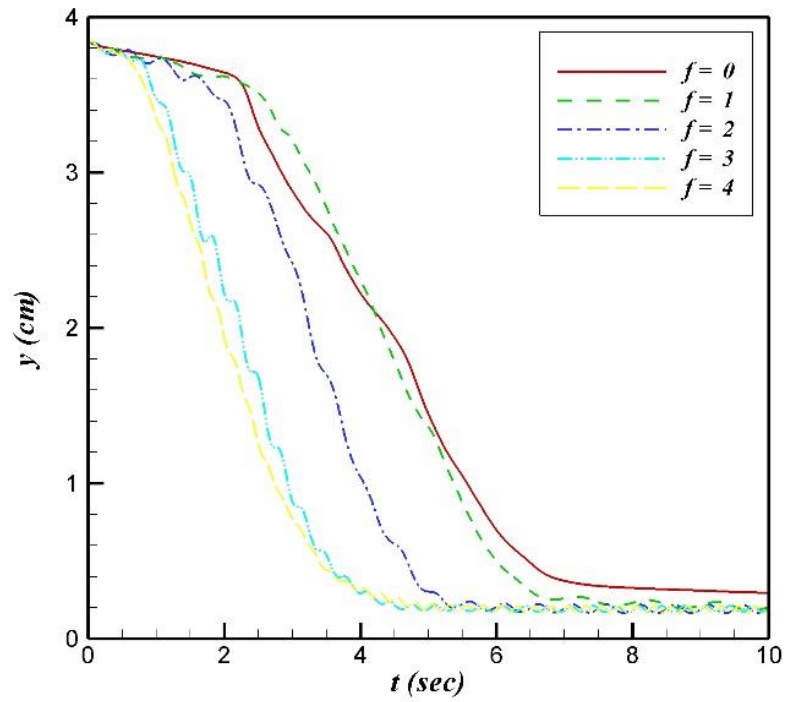


Figure 4. Y-location of the center of gravity of the particles for the yield stress fluid ( $Y_g=0.05$ ).

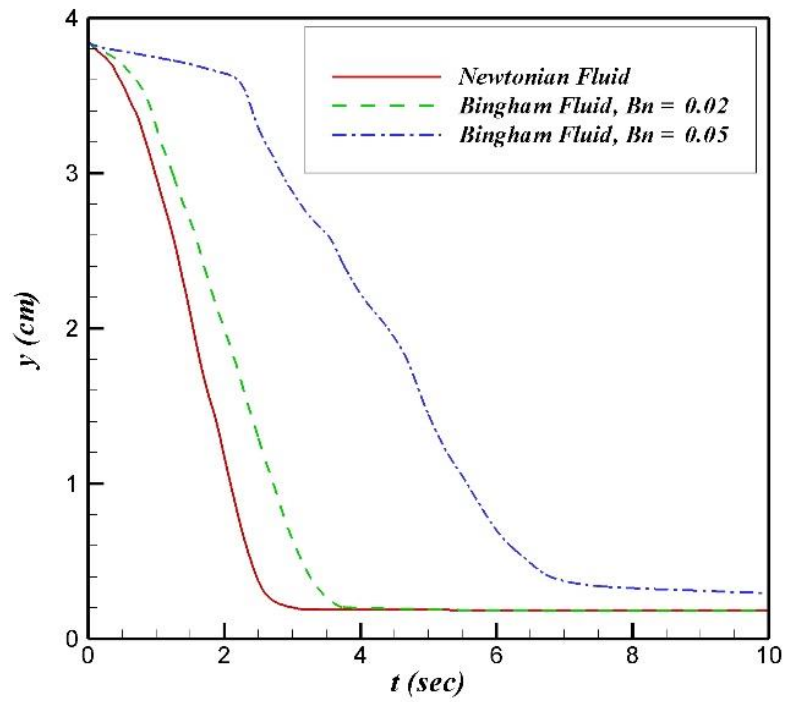
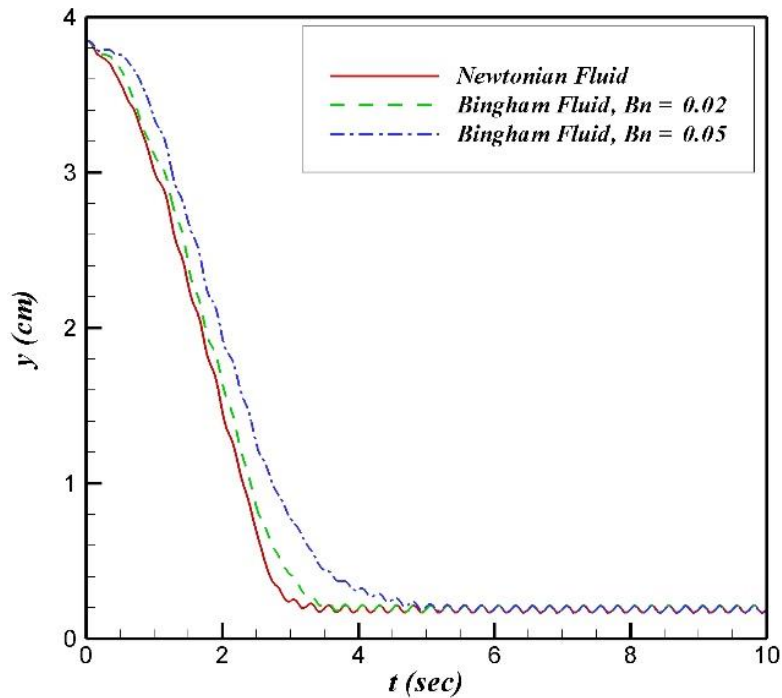


Figure 5. Y-location of the center of gravity of the particles in stationary container.



**Figure 6. Y-location of the center of gravity of the particles in the container with oscillation with  $f=4(1/s)$ .**

### 3. Conclusion

It can be concluded that vibrating the container to speed up the falling of the particles is crucially important in the case of yield stress fluids. Our findings have interesting applications in the vibro-compaction process of the paste in a mold box. For example, it was interestingly shown that, counter-intuitively, the presence of a yield stress may actually help the sedimentation process by vibration. Thus, the presence of yield stress in binder may be the reason why the vibro-compaction is in fact an effective industrial process. Further study is needed to include the huge number of the particles with different shapes and also consider the presence of the gas bubbles in the binder.

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